DESCRIPTION OF VORTEX TURBULENT FLOW OF MIXED LIQUID*

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The article describes a theoretical model of two-dimensional flow of a homogeneous liquid in a cylindrical space delimited by the shell of the mixing tank with a conical bottom, radial baffles and a high-speed axial impeller rotating in a cylindrical draft-tube. By means of the analytical solution of vortex flow equation the mean time flow of turbulent liquid in the vertical cross section of the vessel may be described in the form of Stokes stream function or mean velocity components. The results of theoretical description of the flow are in a good agreement with the experimental values measured by a Pitot direction tube and a hot film anemometer.

Overall characteristics of processes in equipments with mechanical impellers, *e.g.* power input, primary volumetric flow, homogenization time and others, are usually generalized in the form of criterion equations based on the theory of similarity. This approach, however, fails in the case of local characteristics. Therefore, the available data are usually either related to some specific apparatus in which the above characteristics have been determined, or interpreted rather qualitatively (describing certain trends in not fully specified equipment types). As the knowledge of spatial distribution of such quantities is often decisive for understanding of the respective flow phenomena and, simultaneously, their experimental determination is usually more demanding than in the case of global approach, mathematical modelling of hydrodynamics of the liquid contents grows in importance due to the increasing availability of effective computers. In the general case, however, it is necessary to solve a complex system consisting of balance equations for mass, momentum and energy[1,2] for unknown quantities $\bar{w}_r$, $\bar{w}_\varphi$, $\bar{w}_z$, p and ϱ. The difficulty of solving such model is due to the spatial complexity of the given flow realization or to the degree of abstraction from this reality. The purpose of this study is to produce an applicable mathematical model of the field of the mean time components of vortex liquid flow in a vessel with an axial high-speed impeller and internals.

The first attempts to grasp the flow pattern of mechanically stirred liquid had only qualitative character. Schematizing diagrams of liquid circulation were introduced *e.g.* by Rushton and Old-

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shue[3], Porcelli and Marr[4]. The simplest quantitative models postulate potential flow of (ideal) liquid. For instance, the model of the so-called Rankin vortex requires the hyperbolic shape[2] of the radial profile of tangential velocity component $\bar{w}_\varphi$ in the potential flow. This model is often combined with the idea of the so-called vortex core, according to which a certain inner volume of liquid rotates — due to its viscosity — with a constant angular velocity (linear radial profile of $\bar{w}$). The model is expected to be valid especially in case of unbaffled tanks[5,6]. It was however applied also to the mixer with radial baffles by Fořt, Neugebauer and Pastyříková[7] and by Medek and Fořt[8]. It was found that in the liquid flow leaving the axial blade impeller even the axial velocity component $\bar{w}_z$ has a similar profile like the component $\bar{w}_\varphi$, *i.e.* linear and subsequently hyperbolical. The solution of the potential equation of the (non-vortex) flow in other sections of the tank at a similar configuration of the system is given by Fořt, Koza and Gračková[9], and Fořt, Jaroch and Hošťálek[10]. In the former study a numerical method of solution of the given equation was used, maximum error of the calculated streamline function values in the considered area below the impeller being 30%. The authors of the latter study have found a general analytic solution satisfying the potential flow equation in the form of infinite series of particular solutions. This solution, when applied to the upper tank section, *i.e.* to the space above the level of impeller upper blades, showed the error of about 10% with respect to the flow function. Fořt, Obeid and Březina[11] have used somewhat simpler solution of the equation. They applied a particular solution in the form of a simple algebraic function to the mixing system with a turbine impeller and radial baffles. The drawback of their not very complex calculation method consisted in the necessity to divide the described volume rather elaborately into many sub-sections. The model error had then amounted to 30%.

A more realistic assumption for the description of processes in the mixed tank is the existence of vortex liquid flow in the major part of the tank volume (*i.e.* not only within vortex core). Theoretical solution of the two-dimensional vortex liquid flow in a cylindrical vessel published as early as 1884, is quoted by Gray and Matthews[12]. The method has based on the simple dependence of the of liquid particles vorticity on the radial coordinate (direct proportion). The solution of analogous equation for three-dimensional axisymmetrical ideal liquid flow in a vessel without radial baffles and with a turbine impeller placed at the bottom was published by Martynov[13]. The author states a good qualitative agreement of his model with reality and points out the rather schematic nature of the simple model of combined Rankin vortex. For a similar configuration with radial baffles a model of viscous liquid flow was published by Pavlushenko and Kopyleva[14]. Their analytical solution of axisymmetrical vortex flow was in a good agreement with the published data. The axial mean velocity component in systems with a turbine impeller and radial baffles was modelled by Orlov, Tchepura and Tumanov[15]. Having assumed constant viscosity in the whole liquid volume, they derived the formula for calculating radial velocity profile $\bar{w}_z$ (parabolical dependence) in the cross section leading through the centres of axial-radial liquid circulation. The approximate analytical description of circulation liquid flow in the vessel with radial impeller satisfying the continuity equation has been given by Platzer and Noll[16]. In their following work[17] the authors have used an analogous approach for the description of mean flow in tank with axial impeller stating a good agreement of their model with measurement results.

Recently numerous attempts have been made to solve the equations of real liquid motion numerically. Kuriyama and coworkers[18] for instance have used this approach to model high viscous liquid flow in horizontal cross section of a mixing tank with an anchor type impeller. Apart from mean velocity field determination their method also allowed calculation of impeller power input. Numerically, the motion equations for an axisymmetrical isotropically turbulent flow of real Newtonian liquid in a vessel with radial baffles have been solved by Placek[19], who — beside the entire description of mean velocity — also obtained the field of effective values of turbulence. Another example of numerical solution of transport equations is the work of Harvey and Greaves[20], who — in addition to the mean velocity field — have also described the distribution of some turbulent flow characteristics.

THEORETICAL

The object of investigation is the agitated system, schematically represented in Fig. 1. A system of cylindrical coordinates r, φ, z is introduced, with the origin in the intersection of the vessel axis and the plane of the upper base of the conical bottom, which is on the level of the lower edge of the draft-tube and of impeller blades.

To facilitate the description of processes in the tank, the liquid volume may be divided into four simply continuous regions. (Fig. 1). Three of them have a cylindrical shape or a shape of hollow cylinder (region I inside the draft-tube, region IV close to the liquid surface, region III between the draft-tube and vessel walls), while the last one (region II) is in the shape of truncated cone. It is apparent that only this region has an unsuitable shape with respect to the introduced coordinate system (in the system of cylindrical coordinates the oblique bottom wall cannot constitute the so-called coordinate surface), therefore it ought to be excluded from further considerations. Remaining regions I, III and IV shall then be described in a formally identical way based on analytical solution of vortex flow equation.

Let us first formalize the boundaries of the modelled regions. The coordinate surface forming the inner or the outer lateral area shall always be characterized by the value of radial coordinate r_0, r_n, respectively and the coordinate z_0 and z_1. Furthemore, let us assume the fulfilment of the following simplifying conditions

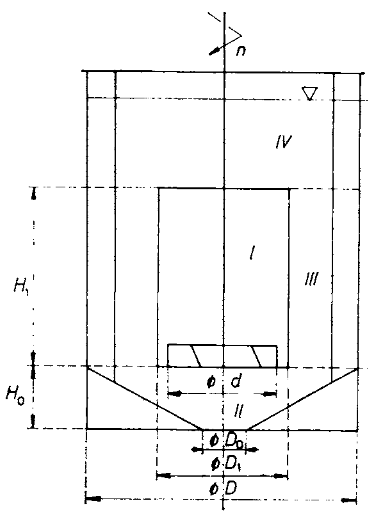

FIG. 1

Mixing equipment with an axial high-speed impeller and internals for the above areas: *a*) The liquid is incompressible and isothermal. *b*) The flow is quasistationary and has a vortex character. *c*) In case of intense turbulent flow the influence of inertial forces on the vortex transport may be neglected. *d*) The flow is symmetrical along the vertical system axis. *e*) The motion of liquid particles takes place in a radial-axial plane.

For an axisymmetrical vortex flow of incompressible liquid with a zero value of tangential velocity component a Navier–Stokes equation in the form

$$\frac{\partial(E^2\psi)}{\partial\tau} - \frac{1}{r}\frac{\partial\psi}{\partial z}\frac{E^2\psi}{r} - \frac{1}{r}\frac{\partial(\psi, E^2\psi)}{\partial(r, z)} = \nu E^2(E^2\psi) \qquad (1)$$

is given by Bird and coworkers[22]. The fulfilment of continuity equation is secured by using Stokes stream function ψ, from which the components of liquid velocity vector $\bar{w} = (\bar{w}_r, 0, \bar{w}_z)$ may be derived according to the relations

$$\bar{w}_r = \frac{1}{r}\frac{\partial\psi}{\partial z}\,; \quad \bar{w}_z = -\frac{1}{r}\frac{\partial\psi}{\partial r}\,. \qquad (2)$$

Under these conditions the vorticity vector $\boldsymbol{\Omega} = \nabla \times \bar{\mathbf{w}}$ has the only non-zero component $\boldsymbol{\Omega} = (0, \Omega_\varphi, 0)$, which can be, using relations (*2*), expressed as follows:

$$\Omega_\varphi = \frac{E^2\psi}{r}\,. \qquad (3)$$

The differential operator E^2 which also appears in Eq. (*1*) has the following form

$$E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\,. \qquad (4)$$

Equation (*1*) is still too complex for a general solution under the given conditions. But the situation may be substantially simplified by the application of assumptions *b*) and *c*). Thus the elements on the left hand side of the equation vanish. Dividing the equation by means of coefficient ν, we obtain a linear differential expression

$$E^2(E^2\psi) = 0 \qquad (5)$$

or, with respect to relation (*3*)

$$E^2(r\,.\,\Omega_\varphi) = 0 \qquad (6)$$

which may be solved analytically.

The boundary conditions of flow models (3) and (6) on those boundaries which are inpenetrable for the liquid flow are summed up in Table I. On these boundaries, always characterized by the constant ψ value (one of the basic properties of this function), the zero value of Ω_φ is always defined. On the other hand, for those parts of region boundary which intersect the liquid flow (it is always one of the cylinder bases) a non-homogeneous boundary condition in the form of the radial profile of ψ (or $\overline{w}_z$) and $\overline{w}_r$ must be introduced.

For the solution of model equations (3) and (6) the method of separation of variables is used[23]. Supposing the particular solution of Eq. (6) can be expected in the form of the product of functions

$$\Omega_{\varphi,i}(r, z) = T_{1,i}(r) \cdot O_i(z) \tag{7}$$

relation (6) may be transformed into two easily integrable ordinary differential equations

$$r^2 T''_{1,i} + r T'_{1,i} - T_{1,i} = 0 \tag{8}$$

and

$$O''_i = 0 \,. \tag{9}$$

If to the solutions thus obtained the subscript $i = 0$ is ascribed, the solution (7) of Eq. (6) may be expressed as

$$\Omega_{\varphi,0} = \left(r + \frac{c_0}{r}\right)(e_0 z + f_0) \,. \tag{10}$$

This particular solution may be satisfactory for the sake of further considerations

TABLE II

Results of application of the boundary conditions on the cylindrical walls of solved regions

Region	C_0	E_0	F_0	$C_i, i = 1, 2, \ldots$
I	0	0	$\frac{\psi(r_n, z)}{r_n^2}$	0
III	$-r_n^2$	0	$-\frac{\psi(r_0, z)}{r_n^2 - r_0^2}$	$-\frac{J_1(k_i \cdot r_0)}{N_1(k_i \cdot r_0)}$
IV	0	0	0	0

If it is substituted into (*3*), we obtain unhomogeneous equation

$$E^2\psi = (r^2 + c_0) . (e_0 z + f_0) , \quad (11)$$

whose solution is the goal of the so far described procedure. The solution of homogeneous equation (*11*), representing a non-vortex (potential) flow, shall be expected in the form

$$\psi_i^*(r, z) = r . T_{1,i}(r) . P_i^*(z) ; \quad i = 0, 1 \ldots . \quad (12)$$

Using relation (*12*) the homogeneous differential equation (*11*) may be separated into two ordinary differential equations and then solved.

The function of the radial coordinate may then be determined in the form of algebraic function

$$T_{1,0}(r) = r + \frac{C_0}{r} ; \quad i = 0 , \quad (13)$$

or cylindrical function[24]

$$T_{1,i}(r) = \mathrm{J}_1(k_i r) + C_i \mathrm{N}_1(k_i r) ; \quad k_i > 0 , \quad i = 1, 2, \ldots \quad (14)$$

(the first index T corresponds to Bessel function index J or to that of Neumann function N). The dependence of solution (*12*) on the axial coordinate is – in the simplest case – linear

$$P_i^*(z) = E_0 z + F_0 ; \quad i = 0 \quad (15)$$

or given by the hyperbolic function

$$P_i^*(z) = E_i \sinh(k_i z) + F_i \cosh(k_i z) ; \quad k_i > 0 ; \quad i = 1, 2, \ldots \quad (16)$$

In order to include also the right hand side of equation (*11*) into this solution, this algebraic function is formally transformed into an infinite series[24] applying the rules of Fourier–Bessel development

$$\Omega_{\varphi,0}(r, z) = \sum_{i=1}^{\infty} T_{1,i}(r) . O_i(z) . \quad (17)$$

Here, function O_i includes linear dependence Ω_φ on the axial coordinate, contained in Eq. (*11*), namely

$$O_i(z) = \omega_i . z + \hat{\omega}_i \quad (18)$$

and function $T_{1,i}$ satisfies relation (*12*).

Analogously as in solving the homogeneous equation, the particular solution of inhomogeneous equation (*11*) will be assumed in the form

$$\psi_i(r, z) = r \,.\, T_{1,i}(r) \,.\, P_i(z)\,; \quad i = 0, 1, \ldots \tag{19}$$

which, having been applied to the equation to be solved, converts it into the form of an ordinary differential equation

$$P_i''(z) - k_i^2 \,.\, P_i(z) = O_i(z)\,; \quad i = 0, 1, \ldots \tag{20}$$

whose solution can be found by means of variation of constants[23] as

$$P_i(z) = P_i^*(z) + k_i^{-1} \,.\, \int_{z_0}^{z} O_i(\xi) \,.\, \sinh\,(k_i \,.\, z - k_i \,.\, \xi)\,\mathrm{d}\xi\,; \quad i = 0, 1, \ldots. \tag{21}$$

Due to the linear character of this differential equation, the linear solution of flow model (*11*) may be written in the form of an infinite sum of particular solutions, where the zero member includes the simplest case of potential flow – piston flow – and is given by the product of functions (*13*) and (*15*), and the following terms are represented by the product of (*14*) and (*21*), *i.e.*

$$\psi(r, z) = \sum_{i=0}^{\infty} \psi_i = \psi_0 + r \,.\, \sum_{i=1}^{\infty} T_{1,i}(r) \,.\, P_i(z)\,. \tag{22}$$

In every member of the solution of flow model (*22*) there are six unknown parameters k, C, F, ω and $\hat{\omega}$ (as for the zero member, the values of k_0, ω_0 and $\hat{\omega}_0$ are *a priori* equal to zero), which are to be determined from the boundary conditions of the solution. The boundary conditions on the vertical coordinate surface $r = r_0$ and $r = r_n$ (Table I) are most likely satisfied, if the value ψ_i ($i = 1, 2, \ldots$) is zero, *i.e.* if we put

$$T_{1,i}(r_0) = T_{1,i}(r_n) = 0\,; \quad i = 1, 2, \ldots \tag{23}$$

and, moreover, if the possible non-zero value of ψ is included in the term ψ_0. As the zero value of parameter E_0 directly follows from the fact that the boundary conditions for ψ are independent on the axial coordinate z, (*24*) must be fulfilled

$$\psi(r_\xi, z) = r_\xi \,.\, T_{1,i}(r_\xi) \,.\, F_0\,, \quad \xi = 0, n\,. \tag{24}$$

The explicit expressions determining the model parameters derived from conditions (*23*) and (*24*), are summed up in Table II. Furthermore, conditions (*23*) implicitly prescribe the progression of values k_i, which, however, can be evaluated by means of a suitable numerical method only after the conditions of the given problems are specified.

The boundary conditions in the bases of the examined region are given, in the case of a cross section through which the liquid passes, by the radial profile of the stream function ψ (or the velocity component $\bar{w}_z$) and $\bar{w}_r$. These are most probably given in the form of an algebraic function, which can be transformed by means of a suitable orthogonalization method. With respect to the boundary conditions at r_0 and r_n (Table I), the Fourier–Bessel development can be used for the radial velocity component

$$\bar{w}_r(r, z_\xi) = \sum_{i=0}^{\infty} \bar{w}_{r,i} = \sum_{i=1}^{\infty} T_{1,i}(r) \, . \, P'_i(z_\xi) \, ; \quad \xi = 0, 1 \tag{25}$$

(according to Table II the term $\bar{w}_{r,0}$ equals to zero). For the axial velocity component the respective development has the properties of the Dini series

$$\bar{w}_z(r, z_\xi) = \sum_{i=0}^{\infty} \bar{w}_{z,i} = - \sum_{i=1}^{\infty} k_i T_{0,i}(r) \, . \, P_i(z_\xi) + \bar{w}_{z,0} \, , \quad \xi = 0, 1 \, . \tag{26}$$

The coefficients in (25) or (26) may then be determined according to the formula[24]

$$\begin{Bmatrix} P'_i(z_\xi) \\ - k_i \, . \, P_i(z_\xi) \end{Bmatrix} = \frac{\displaystyle\int_{r_0}^{r_n} r \, . \begin{Bmatrix} \bar{w}_r \\ \bar{w}_z \end{Bmatrix} . \begin{Bmatrix} T_{1,i}(r) \\ T_{0,i}(r) \end{Bmatrix} . \, \mathrm{d}r}{\displaystyle\int_{r_0}^{r_n} r \, . \begin{Bmatrix} T^2_{1,i}(r) \\ T^2_{0,i}(r) \end{Bmatrix} . \, \mathrm{d}r} \, , \quad i = 1, 2, \ldots \tag{27}$$

TABLE I

Determination of the boundaries of modelled regions and of boundary conditions for ψ and Ω_φ in the case of inpenetrable cross sections

Region	ξ	r_ξ	$\psi(r_\xi, z)$	$\Omega_\varphi(r_\xi, z)$	ζ	z_ζ	$\psi(r, z_\zeta)$	$\Omega_\varphi(r, z_\zeta)$
I	0	0	0	0	0	0	—	—
I	n	$D_1/2$	const. > 0	0	1	H_1	—	—
III	0	$D_1/2$	const. > 0	0	0	0	—	—
III	n	$D/2$	0	0	1	H_1	—	—
IV	0	0	0	0	0	H_1	—	—
IV	n	$D/2$	0	0	1	H	0	0

From thus determined parameter values $P_i(z_0)$, $P'_i(z_0)$, $P_i(z_1)$ and $P'_i(z_1)$, the remaining model parameters E_i, F_i, ω_i and $\hat{\omega}_i$ contained in Eq. (*22*) may be calculated. The function $P_i(z)$ may then be found in the form

$$P_i(z) = \frac{\begin{gathered}[k_i^2 \,.\, P_i(z_0) + O_i(z_0)] \,.\, \sinh(k_i z_1 - k_i z) + \\ + [k_i^2 \,.\, P_i(z_1) + O_i(z_i)] \,.\, \sinh(k_i z - k_i z_0)\end{gathered}}{k_i^2 \,.\, \sinh(k_i z_1 - k_i z_0)} - \frac{O_i(z)}{k_i^2} \,. \qquad (28)$$

For the function $O_i(z)$ the calculation scheme may be — for instance — expressed as

$$\begin{gathered}[2 + k_i \,.\, (z_1 - z_0) \,.\, \sinh(k_i z_1 - k_i z_0) - 2\cosh(k_i z_1 - k_i z_0)] \,.\, O_i(z) = \\ = -k_i^3 \left[(z_1 - z) \,.\, P_i(z_0) + (z - z_0) \,.\, P_i(z_1) + \frac{P'_i(z_1) - P'_i(z_0)}{k_i^2}\right] \sinh(k_i z_1 - k_i z_0) + \\ + k_i^2 \,.\, [-(z_1 - z) \,.\, P'_i(z_0) + (z - z_0) \,.\, P'_i(z_1) + P_i(z_0) + P_i(z_1)] \,. \\ . \cosh(k_i z_1 - k_i z_0) + k_i^2 \,.\, [-(z - z_0) \,.\, P'_i(z_0) + (z_1 - z) \,.\, P'_i(z_1) - \\ - P_i(z_0) - P_i(z_1)] \,. \end{gathered} \qquad (29)$$

Relations (*28*) and (*29*) are satisfactory for regions I and III. In case of region IV the parameter $P'_i(z_1)$ cannot be determined from relation (*27*), as the velocity at the liquid surface is not measured. From the condition of the zero value of Ω_φ (Table I), however, the zero value of the function $O_i(z_1)$ results. Consequently, required value $P'_i(z_1)$ may be determined from (*29*) after a suitable rearrangement.

The so far described solution method demands a separate solution of every region, *i.e.* regardless to the results of modelling of the adjacent region. If we want to interconnect the three solutions, each of them must satisfy the conditions

$$\Omega_\varphi^{\mathrm{I}} = \Omega_\varphi^{\mathrm{IV}} \,;\quad \bar{w}_r^{\mathrm{I}} = \bar{w}_r^{\mathrm{IV}} \,;\quad \bar{w}_z^{\mathrm{I}} = \bar{w}_z^{\mathrm{IV}} \,;\quad z = \mathrm{H}_1 \,;\quad r \in \langle 0;\, D_1/2 \rangle \qquad (30a)$$

or

$$\Omega_\varphi^{\mathrm{III}} = \Omega_\varphi^{\mathrm{IV}} \,;\quad \bar{w}_r^{\mathrm{III}} = \bar{w}_r^{\mathrm{IV}} \,;\quad \bar{w}_z^{\mathrm{III}} = \bar{w}_z^{\mathrm{IV}} \,;\quad z = H_1 \,,\quad r \in \langle D_1/2 \,;\, D/2 \rangle \,, \qquad (30b)$$

where the upper index (Roman numerals) signifies the relation to the given region.

Certain difficulty in comparing individual solutions on the interface of two regions term after term arises due to the existence of the zero term of the series for $\bar{w}_z$ in regions I and III, while in region IV it has a zero value. Here the departing point is again the repetition of Dini development, by means of which series (*26*), which, due to the use of functions T_i^{I} and T_1^{III} has a non vanishing zero term, is transformed into a series without zero term, using the function T_i^{IV}. From conditions (*30*) the values of $P_i(H_1)$

and P_i' (H_1) corresponding to the examined region are obtained, without making experimental examinations in this cross section. Further calculation of the given region may then proceed by a standard above described way. More detailed data about the method are quoted in reference[25].

EXPERIMENTAL

Experiments were carried out in the apparatus represented in Fig. 1. Their results have been used for the determination of the boundary conditions and for estimating the suitability of the equipment for flow modelling. The mixing tank used had diameter $D = 1$ m. The angle of the bottom in the shape of truncated cone was 120° and the diameter of its smaller (lower) base equalled to one sixth of the vessel diameter, *i.e.* 0·17 m. The width of four radial baffles reaching from the bottom to the liquid surface equalled to one tenth of vessel diameter, *i.e.* 0·10 m. The diamater d of a high-speed impeller with six plane blades reached 1/3 or 2/5 of the tank diameter D, *i.e.* 333 or 400 mm. In accordance with the standard[26] the blade angle was 45° and the blade width equal to 1/5 of d. The diameter D_1 of the draft-tube always exceeded the impeller diameter d by 10% – *i.e.* equalled to 366 mm or 440 mm. The draft-tube length H_1 was in both cases 667 mm. The liquid surface height reached the value $H = 1$ m in the cylindrical part of the tank. In all experiments tap water was used. The charge volume was 0·86 m^3.

For determination of the so-called mean flow of turbulent liquid in the chosen profiles a five hole Pitot tube was used. This combined pressure sensor (already described by Krátký and coworkers[27]) looks like a small cylinder 3·5 mm in diameter with the entering end in the shape of a foursided truncated pyramid with vertex angle 90°. The centres of individual probe holes are in two perpendicular planes intersecting in the axis of the middle hole. The holes were connected with pressure gauges (inclined manometers under the angle 30°) by means of polyethylene tubings in a way securing the time stabilization of data. The measurement results consisted especially of the data of the angles of the mean velocity direction $\bar{w}$. After obtaining these data the measurement of the mean velocity $\bar{w}$ by thermoanemometric method followed. Here, the electrically heated sensor of constant temperature is exposed to the cooling effect of liquid stream. In dependence on the velocity of flow the rate of heat transfer between the sensor and the medium can also be specified. And knowing the heating voltage of the probe, we may then assess the flow velocity (non-linear dependence). In the experiments thermoanemometer[28] DISA was used, consisting of feed unit 55M05, main unit 55M01 and standard bridge 55M10. As a sensor a wedge-shaped probe was used, consisting of a nickel film 1 mm in length and 0·2 mm in width protected from the environment by a quartz layer about 2 μm thick (see[29]). The output voltage signal of the thermoanemometer was processed by means of an additional electronic equipment – linearizer[30], its respective transformation function for the given probe was adjusted by means of callibration measurements. Finally, the mean value of linearized voltage signal was recorded by the digital voltmeter DISA 55D31, the proportionality constant between voltage and mean velocity value $\bar{w}$ was given by the choice of the terminal amplification of the linearizer.

The motion of the sensors in question, *i.e.* the Pitot tube or the thermoanemometer probe in the liquid volume was realized by means of a special equipment, constructed especially for the purpose of their precise and repeated adjustment. From the analysis of possibilities of different experimental methodologies we have estimated that the maximum error of the assessment of velocity direction would in most cases be less than 10° and the relative error would not exceed 5 to 10%.

RESULTS AND DISCUSSION

Choice of Equipment Configuration

The examined equipment is characteristic for a more complex inner configuration (Fig. 1) than is usual in descriptions of mixing processes. The purpose of internals is to attain possibly the same reological conditions in the whole tank volume, which plays an important part in a number of processes, *e.g.* crystallization from solutions and others. The configuration is based on the results of preceding research[33,34], in the course of which the dependence of the impeller volumetric flow rate and power input on the type of draft-tube and its position in the tank, on impeller size, its type and its position with respect to the draft-tube *etc.* was examined. Especially the last mentioned geometrical parameter proved to be very important (the maximum of impeller pumping effect in the lowest part of the draft-tube).

Adequacy of Initial Assumptions

The fulfilment of the initial assumptions of the model simplifying the given problems may be expected due to the physical properties of water as model charge (incompressible liquid), the use of the high-speed impeller, constant frequency of revolutions, symmetrical tank configuration and installation of baffles. Similar simplifications are currently adopted for the sake of modelling the mixing process. A more detailed explanation, however, is necessary in case of assumption *c*) corresponding to the so-called Stokes simplification for creeping flow. In adopting it even for the case of fully turbulent flow, we were following a notion about prevailing influence of turbulence on transfer of properties. Kotchin and coworkers[1] for instance state that the so called turbulent viscosity under fully turbulent flow regime may exceed the value of molecular viscosity by 5 or 6 orders. In other words, an analogy is supposed between vortex propagation in two totally different flow patterns (creeping *vs* turbulent flow).

Boundary Conditions of the Model

The introduction of boundary conditions on the boundaries which are penetrable for the streaming liquid depends in the given model on adequate experiments. For certain cross sections the model requires knowledge of mostly radial profiles of velocity components $\bar{w}_r$ and $\bar{w}_z$. As the discrete form of experimental data is unsuitable for their integration into the mathematical model, the individual profile sections have gradually been replaced by straight lines

$$\bar{w}_r = \alpha_r \, . \, r + \hat{\alpha}_r \, , \quad r \in \langle r_\xi ; \, r_{\xi+1}) \, ; \quad \xi = 0, 1, \ldots \tag{31}$$

(and, analogously, for $\bar{w}_z$). The range and number of sections corresponded to the individual character of the measured profile of the given velocity component in the respective base of the investigated region. To the obtained relation (*31*) formula (*27*) was applied, whose numerator, however, was transformed into the sum of integrals for individual linear sections of the profile. The example of the described replacement by measurement of the given velocity components profile by means of a broken line is shown in Fig. 2. Similar procedure was chosen also in the remaining cases. The exception was the lower base of region I, where experiments with the equipment were not realized due to the lack of space. Analogously, with the results in the region of the draft-tube inlet and according to the quoted data[7] for draft-tube configuration, we have defined the so-called piston flow, *i.e.* the constant radial profile of $\bar{w}_z$ at zero value of $\bar{w}_r$, for the plane of liquid inflow into the impeller.

Model Calculation and Modelling Results

The use of the model is conditioned by the calculation of functional values of cylindrical functions J_0, J_1, N_0 and N_1. In this case, polynomial approximations given by Abramovitz and Stegun[31] have been applied. The zero points (roots) of cylindrical function (*23*) have been obtained by means of Newton tangent method[23]. In solving relation (*27*) the introduction of boundary conditions in equation (*31*) has led to the necessity of calculating the values of further special functions given by integrals

$$\int_0^x J_0(\xi)\,d\xi \quad \text{or} \quad \int_0^x N_0(\xi)\,d\xi\,; \qquad (32)$$

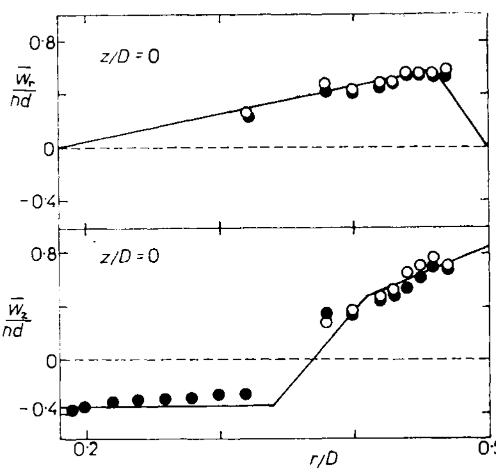

FIG. 2

An example of gradual replacement of the experimentally obtained radial profile of velocity components $\bar{w}_r$ and $\bar{w}_z$ by linear sections (d/D 0·333; ● n 210 min^{-1}; ○ n 150 min^{-1})

For values $x \leqq 8$ the integrated definition relations for J_0 and N_0 (see[24]) were used, for $x > 8$ the method of asymptotic integral development into power series was applied, which was based on the repeated use of integration per partes[32]. The error of these numerical operations was mostly smaller than 10^{-7}. A quick convergence of series (*22*), representing the solution of the chosen model made it possible to finish the solution even with a small number of its terms. In solving the model region by region, 10 to 12 terms were used and in solving the whole cylindrical space of the tank the number of terms was reduced by half to speed up the calculation. This was carried out by means of the minicomputer WANG MVP programmed in the language Basic – 2.

The results of flow modelling for the smaller of the impellers used ($d/D = 0{\cdot}333$) are represented in the form of streamlines in Figs 3 and 4. The former figure describes a situation following from the separate description of individual regions I, III and IV, the latter represents the global solution of the cylindrical part of the tank, *i.e.* the interconnection of solutions for the individual regions. Fig. 5 gives an example of comparison of the results of the modelling procedures with data derived from the experi-

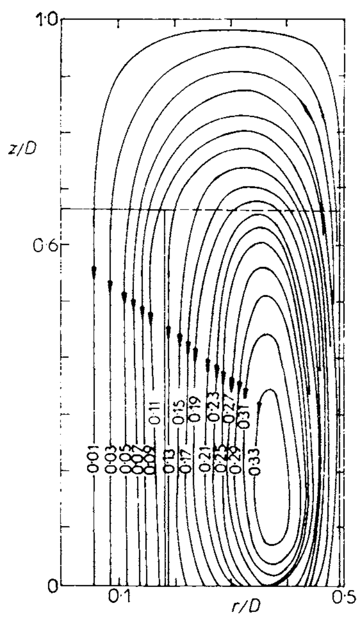

FIG. 3

Modelling mean liquid flow region by region for impeller of d/D 0·333

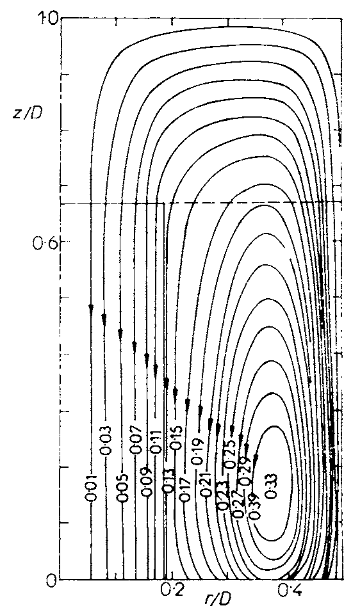

FIG. 4

Modelling mean liquid flow for the cylindrical part of the tank as a whole for impeller d/D 0·333 mental results. The course of individual curves significes a good quantitative agreement of the model with reality. The analysis of the best examined region III has implied that the error of determination of stream function values by the given model in the ascending flow at the vessel wall was in most cases smaller than 20% for both impeller sizes. Certain deterioration could be noticed only at the bigger impeller ($(d/D = 0{\cdot}4)$, when, in case of modelling the whole of the cylindrical part of the tank, the maximum error of the stream function determination in the upper base of region III near the vessel wall amounted to 50%. In comparison with the maximum value measured in this region the error was just about 30%.

CONCLUSION

The described method of modelling of the mean liquid flow proved fully applicable for the description of the behaviour of homogeneous liquid in the tank with a draft-tube, except for the regions of conical bottom and impeller rotor. Even in case of relatively complex space division the requirements on experimental investigation may be reduced to a single cross section (lower base of region III). The advantage of the model consists in an explicit form of the obtained solution. The described concenption is able of further development especially as far as the introduction of more realistic boundary conditions of the boundaries intersecting the liquid flow and apparently even the reassessment of some simplifying conditions are concerned. Even in its present state the model may be applied on other equipments with similar flow conditions.

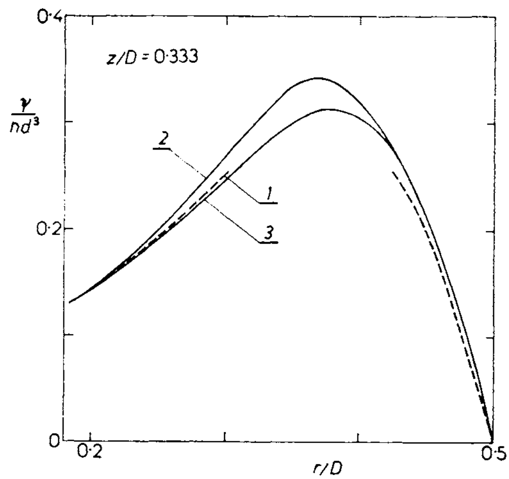

FIG. 5

Radial profile of the stream function in region III, assessed from the experimental data (curve *1*), with the results of the modelling of region III (curve *2*) and with the results of modelling the whole of cylindrical part of the tank (curve *3*)

LIST OF SYMBOLS

C parameter of vortex flow model
c_0 parameter of solution of Eq. (*6*)
D mixing tank diameter (m)
D_0 diameter of lower base of conical bottom (m)
D_1 draft-tube diameter (m)
d impeller diameter (m)
E parameter of vortex flow model
E^2 differential operator defined by Eq. (*4*)
e_0 parameter of solution of Eq. (*6*)
F parameter of vortex flow model
f_0 parameter of solution of Eq. (*6*)
H liquid height in cylindrical part of tank (m)
H_0 height of conical bottom (m)
H_1 height of draft-tube (m)
J_p cylindrical function of first kind (Bessel) of index p
k_i i-th root of transcendental function $T_1(k)$
N_p cylindrical function of second kind (Neumann) of index p
n impeller rotation frequency (s^{-1})
O_i course of dependence of Ω_φ on axial coordinate
P course of dependence of ψ on axial coordinate
p pressure (Pa)
r radial coordinate (m)
r_0 inner diameter of hollow cylinder (m)
r_n outer diameter of (hollow) cylinder (m)
T_p course of dependence of $\overline{w}_r(p = 1)$ or $\overline{w}_z(p = 0)$ on radial coordinate
$\overline{\mathbf{w}}$ liquid mean velocity vector ($m\,s^{-1}$)
x general variable
z axial coordinate (m)
z_0 position of cylinder lower base (hollow cylinder) (m)
z_1 position of cylinder upper base (hollow cylinder) (m)
α parameter of Eq. (*31*)
$\hat{\alpha}$ parameter of Eq. (*31*)
ζ general variable
ν eddy viscosity ($m^2\,s^{-1}$)
ξ general variable
ϱ density ($kg\,m^{-3}$)
φ tangential coordinate
ψ Stokes stream function (m^3, s^{-1})
$\boldsymbol{\Omega}$ vorticity vector (s^{-1})
ω_i parameter of vortex flow model defined by Eq. (*18*)
$\hat{\omega}_i$ parameter of vortex flow model defined by Eq. (*18*)

Subscripts

j addition index (non-negative whole numbers)
i projection into j-th direction
r radial component

z axial component
ζ point on axial coordinate
φ tangential component
ξ point on radial coordinate

Superscripts

I for region I
III for region III
IV for region IV
* solution of homogeneous partial differential equation

REFERENCES

1. Kotchin N. E., Kibel I. A., Roze N. V.: *Teoretitcheskaya Gidromekhanika.* Gos. izd. fiz.-mat. lit., Moscow 1963.
2. Batchelor G. K.: *An Introduction to Fluid Dynamics.* Cambridge University Press, Cambridge 1970.
3. Rushton J. H., Olshue J. Y.: Chem. Eng. Progr. *49*, 161, 167 (1953).
4. Porcelli J. V., Marr G. R.: Ind. Eng. Chem. Fundam. *1*, 172 (1962).
5. Strek F.: *Míchání a míchací zařízení.* Published by SNTL, Prague 1977.
6. Nagata S.: *Mixing — Principles and Applications.* Wiley, New York 1975.
7. Fořt I., Neugebauer R., Pastyříková M.: This Journal *36*, 1769 (1971).
8. Medek J., Fořt I.: This Journal *46*, 959 (1969).
9. Fořt I., Gračková Z., Koza V.: This Journal *37*, 2371 (1971).
10. Fořt I., Jaroch O., Hošťálek M.: This Journal *42*, 3555 (1977).
11. Fořt I., Obeid A., Březina V.: This Journal *37*, 226 (1982).
12. Gray A., Mathews G. B.: *A Treatise on Bessel Functions and Their Applications to Physics* (Russian translation), Moscow 1953.
13. Martynov Ju. V.: Teor. Osn. Khim. Technol. *14*, 575 1980).
14. Pavlushenko I. S., Kopyleva B. B.: Teor. Osn. Khim. Technol. *11*, 107 (1977).
15. Orlov V. A., Tchepura I. V., Tumanov Yu. V.: Teor. Osn. Khim. Technol. *10*, 261 (1976).
16. Platzer B., Noll G.: Chem. Tech. (Leipzig) 648 (1981).
17. Platzer B., Noll G.: Chem. Tech. (Leipzig) *34*, 642 (1982).
18. Kuriyama M., Inomata H., Arai K., Saito S.: AIChE J. *28*, 385 (1982).
19. Placek J.: *Thesis.* Microbiological Institute of Czechoslovak Academy of Sciences, Prague 1980.
20. Harvey P. S., Greaves M.: Trans. Inst. Chem. Eng. *60*, 195, 201 (1982).
21. Günkel A. A., Weber M. E.: AIChE J. *21*, 931 (1975).
22. Bird R. B., Steward W. E., Lightfoot E. N.: *Transport Phenomena.* Wiley, New York 1965.
23. Korn G. A., Korn T. M.: *Mathematical Handbook.* McGraw-Hill, New York 1968.
24. Korenyev B. G.: *Vvedeniye v Teoriyu Beselevych Funktsii.* Nauka, Moscow 1971.
25. Hošťálek M.: *Thesis.* Prague Institute of Chemical Technology, Prague 1982.
26. Standard No 6910. VÚCHZ Chepos, Brno 1969.
27. Krátký J., Fořt I., Kroužilová Z., Drbohlav J.: Sb. Vys. Šk. Chemicko-Technol. Praze *K5*, 33 (1971).
28. Description of the DISA 55M System. DISA Elektronik A/S, Herlev 1971.
29. DISA-Probe Manual. DISA Elektronik A/S. Skovlunde 1976.
30. Linearizer DISA 55 M 25. DISA Elektronik A/S. Skovlunde 1978.

31. Abramovitz M., Stegun I. A.: *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*. NBS (USA) 1964.
32. Rektorys K.: *Přehled užité matematiky*. Published by SNTL, Prague 1973.
33. Fořt I., Vlček J., Kudrna V.: Chem. Prům. *25/50*, 449 (1975).
34. Vlček J., Fořt I., Lukešová S.: Chem. Prům. *29/54*, 169 (1979).

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